WARING
SUMMER MATH REVIEW PACKET 2022

Hi incoming **Calculus AP** students!

This is the **required Summer Math Packet** for students whose registration for 2022-23 Calculus AP class has been approved.

The purpose of this math course is to learn about the beauty and utility of Calculus as a discipline, but also to prepare you to take the AP Calculus AB exam in May. Advanced Placement classes cover college-level content during high school at an accelerated pace.

In this summer review packet you will be asked to review and solidify understanding of Limits and Continuity from Unit 1 of the AP Calculus curriculum (much of which was covered in your Introduction to Calculus class this spring), as well as a few important Precalculus ideas. You will be assessed on your understanding of Unit 1's learning objectives in mid-September. We will then move on to Unit 2 in September. To help you out, Page 2 contains a hyperdoc with online resources for you to use to review. Answers to problems are provided at the end on page 11.

The reason for this accelerated timeline is so that we will cover all units (Units 1 - 8) by April and allow time for global review and AP Exam Preparation prior to the Exam. The pace of this course will be rapid, covering approximately one unit a month, in order to stay on pace. You will be expected to put in significant time and effort outside of class. I will be available one Focus/Flex period a week for Problem-Solving review sessions on current material which you may want to prioritize.

If you have ANY questions or problems or just need a bit of help, please feel free to email me at jsullivan@waringschool.org.

Happy summering!!!

Joan

Name ________________________________

Your Task this summer:

1. REVIEW material on Limits and Continuity - Using Resource Links below and your Notes from the Spring
2. Review the UNIT CIRCLE!!!! see Precalculus Review resource below.
3. PREPARE SOLUTIONS to the problems in this packet (BOTH Part 1 – quick Precalculus Review and Part 2 - Limits and Continuity) and bring your questions to the first week of class.

HYPERDOC for Mastery and Review
Calculus AP - Unit 1 Limits and Continuity

Objective	**Instructional Resources**
Precalculus Review (important for Calculus)	Functions and Properties Point-Slope Form of Linear Equation Secant Line Slope (average rate of change) Interval Notation Trigonometry Review - Unit Circle Graphing Calculator Skills for Calculus (TI Family)
Overview of AP Calc Unit 1	***Open source TEXT for UNIT 1*** PDF link here (if you prefer a text) ***MUST WATCH Videos*** Overview of Unit 1 (in 10 minutes) Overview of Unit 1 (all in one hour) Other:Khan Academy (lots of videos/ searchable)
Limits: Notation, Graphical, Numerical, One-Sided (1.1 - 1.4)	Playlist Choose from Videos # 16- 18, 6, 11
Limits: Algebraic techniques (1.5 - 1.7, 1.9)	Playlist Choose from Videos # 1- 5, 22
Continuity and Discontinuity (1.10 - 1.13)	Playlist Choose from Videos # 16- 18, 23
Asymptotes and Infinity (1.14-1.15)	Playlist Choose from Videos # 6 - 8
Intermediate Value Theorem (1.16)	Playlist Choose from Videos # 20

Part 1: Precalculus Review Problems

Directions: See the HYPERDOC on page 2 for resources on MASTERY & REVIEW of these topics, definitions, and notation. Complete solutions to problems 1 – 3, and then review material in problem 4. Answers - not solutions - are provided at the end, so you may check your work. Make sure you understand the solution behind each answer. NOT CALCULATOR ACTIVE.

1. Let's open with a quick Algebra "correct the mistake" activity.

True or false. If false, change what is underlined to make the statement true.

a. $\left(x^3\right)^4 = x^{\underline{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\underline{\frac{3}{2}}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3} + 2\sqrt{x-3} = 5\sqrt{x-3}$ T F

2. Given the **functions** $f(x) = x^2 - 4x \;\; and \;\; g(x) = 5 - x$, find each of the following:

a) f(g(-1))

b) g(f(-1))

c) $\frac{f(x)-f(3)}{x-3}$

d) $g^{-1}(-1)$

3. State the **domain** of the following:

a) $f(x) = \sqrt{(x+2)}$

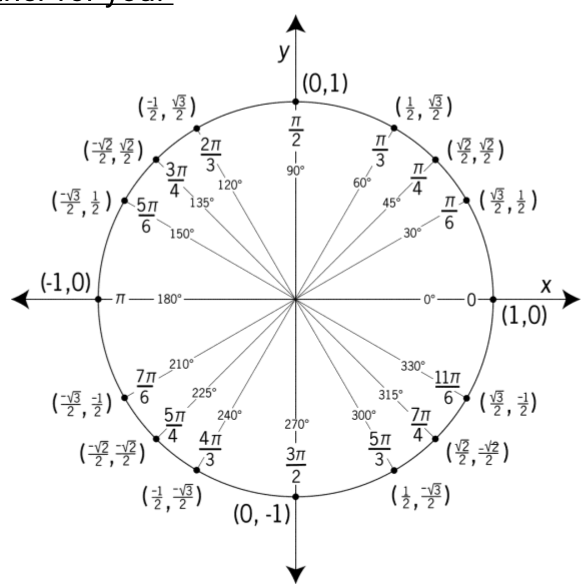
b) $g(x) = \frac{x+2}{x^2-4}$

4. Trigonometry Facts Review

Take time this summer to review the values of sine, cosine, and tangent in the unit circle of the "cardinal angles" in radians. You may use whatever method helps you. All the information you are expected to know is in this unit circle. <u>Having these committed to memory will help calculus run much smoother for you.</u>

Recall that each point on the unit circle $(x, y) = (cos\ \theta, sin\ \ \theta)$ and $x^2 + y^2 = 1$.

θ	0° (or) 0	30° (or) $\frac{\pi}{6}$	45° (or) $\frac{\pi}{4}$	60° (or) $\frac{\pi}{3}$	90° (or) $\frac{\pi}{2}$
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not Defined



Check your answers at the end. Then move on to Part 2: Review of Limits and Continuity.

PART 2 Intro to Calculus Review
LIMITS & CONTINUITY UNIT 1 REVIEW PACKET

__Important Topics:__ ***See the HYPERDOC for resources on MASTERY & REVIEW of these topics, definitions, and notation.***

- Limits
- Limit laws, left and right hand limits, limits at infinity
- Continuity
- Intermediate Value Theorem

Directions: Test your understanding by completing these problems. Write up your own solutions, either on a printout of this packet or on separate sheets (number each problem clearly). Answers - not solutions - are provided at the end, so you may check your work. Make sure you understand the solution behind each answer. You will be tested on this material the second week of classes. NOT CALCULATOR ACTIVE.

1. $\lim_{x \to 0} \frac{3x^5 + 12x^2}{5x^4 + 6x^2}$ A) 0 B) $\frac{1}{2}$ C) 2 D) DNE

2. $\lim_{x \to 3^-} \frac{4 - 3x}{x - 3}$ A) 0 B) ∞ C) $-\infty$ D) -3

3. Let f be the function that is continuous on the closed interval $[3, 5]$ with $f(3) = 9$ and $f(5) = 22$. Which of the following is guaranteed by the Intermediate Value Theorem?

   A) $f(x) = 11$ has at least one solution in the open interval $(3, 5)$.

   B) $f(4) = 13$

   C) $f(x)$ has at least one zero in the open interval $(3, 5)$.

   D) f attains a maximum value on the open interval $(3, 5)$.

4. If $f(x) = 3x^2 + 4x$, then $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

   A) $6x$

   B) $6x + 4$

   C) 0

   D) DNE

$$g(x) = \begin{cases} \sin\dfrac{x\pi}{3}, & x < 2 \\ x\sqrt{2}, & x = 2 \\ \dfrac{x\sqrt{3}}{3x-4}, & x > 2 \end{cases}$$

5. For the function above, which of the following would be the reason(s) why the function, $g(x)$ is not continuous at $x = 2$?

I. $g(2)$ is undefined II. $\lim_{x \to 2} g(x)$ does not exist III. $\lim_{x\to 2} g(x) \neq g(2)$

A) III only

B) II only

C) I and II only

D) II and III only

6. $\lim_{x\to \infty} \frac{2x^2-4x+8x^3}{3x^3-5x^2+7}$ A) ∞ B) $-\infty$ C) $\frac{8}{3}$ D) $\frac{2}{3}$

7. $\lim_{x\to -\infty} \frac{5-3x}{\sqrt{x^2+3}}$ A) ∞ B) $-\infty$ C) -3 D) 3

8. Given the function $H(x) = \begin{cases} x^2 - 3x, & x < 2 \\ 2x - 5, & x \geq 2 \end{cases}$. Which of the following statements is/are true?

I. $\lim_{x\to 2^-} H(x) = -2$ II. $\lim_{x \to 2} H(x)$ exists III. $H(x)$ is continuous at $x = 2$

A) I only B) II only C) I and II only D) None of these

<u>TRUE or FALSE Statements</u>. *If your response is "FALSE", explain why the statement is FALSE. If your response is "TRUE", write TRUE.*

9. $\lim_{x\to 4}\left(\frac{2x}{x-4}-\frac{8}{x-4}\right)=\lim_{x\to 4}\frac{2x}{x-4}-\lim_{x\to 4}\frac{8}{x-4}$

10. $\lim_{x\to 1}\left(\frac{x^2+6x-7}{x^2+5x-6}\right)=\frac{\lim_{x\to 1}(x^2+6x-7)}{\lim_{x\to 1}(x^2+5x-6)}$

11. $\lim_{x\to 1}\frac{x-3}{x^2+2x-4}=\frac{\lim_{x\to 1}(x-3)}{\lim_{x\to 1}(x^2+2x-4)}$

12. If $\lim_{x\to 5}f(x)=2$ and $\lim_{x\to 5}g(x)=0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist.

13. If $\lim_{x\to 5}f(x)=0$ and $\lim_{x\to 5}g(x)=0$, then $\lim_{x\to 5}[f(x)/g(x)]$ does not exist.

14. If $\lim_{x\to 6}f(x)g(x)$ exists, then the limit must be $f(6)g(6)$.

15. If p is a polynomial, then $\lim_{x\to b}p(x)=p(b)$

16. If $\lim_{x\to 5}f(x)=\infty$ and $\lim_{x\to 5}g(x)=\infty$, then $\lim_{x\to 5}[f(x)-g(x)]=0$.

17. If the line $x=1$ is a vertical asymptote of $y=f(x)$, then f is not defined at 1.

18. If $f(1)>0$ and $f(3)<0$, then there exists a number c between 1 and 3 such that $f(c)=0$.

19. If f is continuous at 5 and $f(5)=2$ and $f(4)=3$, then $\lim_{x\to 2}f(4x^2-11)=2$.

20. $\lim_{x\to 0}\frac{|x|}{x}=1$

21. If $\lim_{x\to c}f(x)=L$, then $f(c)=L$.

22. If f is undefined at $x=c$, then the limit of $f(x)$ as x approaches c does not exist.

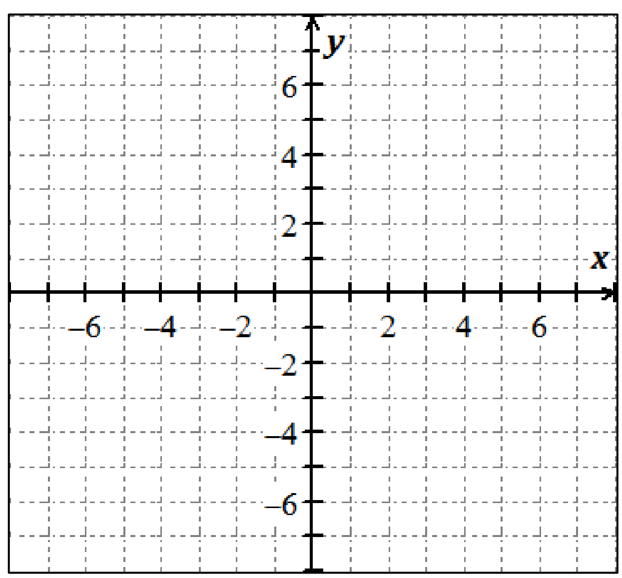
23. Sketch the graph of an example of a function f that satisfies **<u>all</u>** of the following conditions (you may use a piecewise defined function):

$\lim_{x\to 0^+}f(x)=-2$ $\quad\lim_{x\to 0^-}f(x)=1$

$f(0)=-1$ $\quad\lim_{x\to\infty}f(x)=3$

$\lim_{x\to 2^-}f(x)=\infty$ $\quad\lim_{x\to 2^+}f(x)=-\infty$

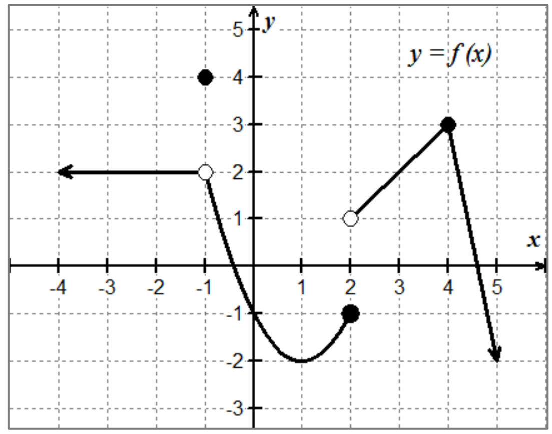
$\lim_{x\to-\infty}f(x)=4$



24. Use the graph of $f(x)$ shown in the figure at the right. Let $g(x) = \sqrt{4 + x^2}$. Evaluate the limits, if they exist.

A.) $\lim_{x \to 2^+} [f(x) + 5\,g(x)]$ B.) $\lim_{x \to 1} [f(x) \cdot g(x)]$

C.) $\lim_{x \to 2^-} \frac{f(x)}{g(x)}$ D.) $\lim_{x \to 4} [g^2(x)]$



25. Find the values of k and m so that the function below is continuous for all reals.

$$f(x) = \begin{cases} x^2 - kx + 3, & x < -2 \\ 2x - 3, & -2 \le x \le 3 \\ 4 - 2m, & x > 3 \end{cases}$$

26. The Intermediate Value Theorem states that given a continuous function f defined on the closed interval $[a, b]$ for which 0 is between $f(a)$ and $f(b)$, there exists a point $x = c$ between a and b such that

A. $c = a - b$

B. $f(a) = f(b)$

C. $f(0) = c$

D. $f(c) = 0$

27. **Limits at Infinity and Infinite Limits**

Function	Limit as $x \to a$	Limit as $x \to \pm\infty$
A.) $f(x) = \|x\|$	$\lim_{x \to 0} f(x)$	$\lim_{x \to -\infty} f(x)$
B.) $f(x) = \frac{\|x\|}{x}$	$\lim_{x \to 0} f(x)$ $\lim_{x \to -5} f(x)$	$\lim_{x \to \infty} f(x)$
C.) $f(x) = 6$	$\lim_{x \to 2} 6$	$\lim_{x \to -\infty} 6$
D.) $f(x) = \cos x$	$\lim_{x \to 0} f(x)$	$\lim_{x \to \infty} f(x)$
E.) $f(x) = \frac{1}{x}$	$\lim_{x \to 0} f(x)$	$\lim_{x \to \infty} f(x)$ $\lim_{x \to -\infty} f(x)$
F.) $f(x) = \frac{x+4}{x-2}$	$\lim_{x \to 1} f(x)$ $\lim_{x \to 2} f(x)$	$\lim_{x \to \infty} f(x)$ $\lim_{x \to -\infty} f(x)$
G.) $f(x) = \frac{x-2}{x^2-4}$	$\lim_{x \to 0} f(x)$ $\lim_{x \to -2} f(x)$ $\lim_{x \to 2} f(x)$	$\lim_{x \to \infty} f(x)$ $\lim_{x \to -\infty} f(x)$
H.) $f(x) = \frac{x^2-4}{x-2}$	$\lim_{x \to 2} f(x)$ $\lim_{x \to 0} f(x)$	$\lim_{x \to \infty} f(x)$ $\lim_{x \to -\infty} f(x)$

FREE RESPONSE STYLE (No answer provided – do your best!):

28. Given the functions $f(x)$ and $g(x)$ stated below, answer the following questions.

$$f(x) = \begin{cases} \dfrac{|2-x|}{2-x} & x < 2 \\ 5x + 3k & x \geq 2 \end{cases} \qquad\qquad g(x) = \frac{x^2 - 25}{x + 2}$$

A. Determine the value of ***k*** such that the function $f(x)$ is continuous over the set of real numbers. (2 points)

B. For the value of ***k*** found in (A), apply the "three-part" definition of continuity to confirm that the function $f(x)$ is continuous. (4 points)

C. Explain why the <u>Intermediate Value Theorem</u> <u>**does not apply**</u> for guaranteeing that a zero exists for the function $g(x)$ over the given interval $[0, 4]$. (3 points)

Part 1 Answers:

1A. T	1B. F	1C. F	
1D. F	1E. T	1F. F	
2A. 12	2B. 0	2C. $\frac{1}{6}(x^2 - 4x + 3)$	2D. 6
3	A. $\{x \mid x \geq 0\}$	B. $\{x \mid x \neq -2,2\}$	

Part 2 Answers: Thank you © FlamingoMath for these problems!

1. C		
2. B		
3. A		
4. B		
5. D		
6. C		
7. D		
8. A		
True/False		
9. F	16. F	
10. F	17. F	
11. T	18. F	
12. T	19. T	
13. F	20. F	
14. F	21. F	
15. T	22. F	
23.	Student answers will vary	
24.	A) $1 + 10\sqrt{2}$	B) $-2\sqrt{5}$
	C) $-\frac{\sqrt{2}}{4}$	D) 20
25.	$k = -7$	$m = \frac{1}{2}$
26.	D	
27A.	0	∞
27B.	DNE, -1	1
27C.	6	6
27D.	1	DNE
27E.	DNE	0, 0
27F.	-5, DNE	1, 1
27G.	$\frac{1}{2}$, DNE, $\frac{1}{4}$	0, 0
27H.	4,2	$\infty, -\infty$
27. Free Response Style. No answers provided.		