WARING
SUMMER MATH REVIEW PACKET 2023

Hi incoming **Calculus AP** students!

This is the **required Summer Math Packet** for students whose registration for 2023-24 has been approved

The purpose of this math course is to learn about the beauty and utility of Calculus as a discipline, but also to prepare you to take the Advanced Placement "Calculus AB" exam in May. Advanced Placement classes cover college-level content during high school at an accelerated pace.

In this summer review packet you will be asked to review and solidify ideas from Algebra and Precalculus, with particular importance to Calculus. There are problems for you to solve and resources for you, if you need review or get stuck. Bring the completed problems to class during the first week. There will be an in-class assessment at the end of that week. See page 2 of this packet which contains a hyperdoc with online resources for you to use to review. Answers to problems are provided at the end. Please check your work and revise your solutions if needed.

The reason for this accelerated timeline is so that we will cover all units (Units 1 - 8) in the Calculus AB curriculum by April and allow time for global review and AP Exam Preparation prior to the Exam. The pace of this course will be rapid, covering approximately one unit a month, in order to stay on pace. You will be expected to put in significant time and effort outside of class. I will be available one Focus/Flex period a week for Problem-Solving review sessions on current material which you may want to prioritize.

If you have ANY questions or problems or just need a bit of help, please feel free to email me at jsullivan@waringschool.org.

Happy summering!!!

Joan

Name _____

Your Task this summer:

1. REVIEW important ideas from PRECALCULUS and ALGEBRA.
2. REVIEW and MEMORIZE (again!) the UNIT CIRCLE.
3. PREPARE SOLUTIONS to the problems in this packet and bring your questions to the first week of class. We will have a test on these ideas in September.

SEE RESOURCES BELOW TO HELP YOU!

HYPERDOC for Mastery and Review
Calculus AP - Precalculus Review & Unit 1 Limits (notation & graphs)

Objective	Instructional Resources
Precalculus Review (important for Calculus)	• Functions and Properties • Point-Slope Form of Linear Equation • Secant Line Slope (average rate of change) • Interval Notation • Factoring Basics • Factoring Quadratics • Trigonometry Review - Unit Circle • Graphing Calculator Skills for Calculus (TI Family) • Math is Fun visual dictionary
Introduction to Calculus & Limits	*Could you Discover Calculus yourself?* Essence of Calculus *(17 minutes - challenging & interesting)* *Introduction to Limits* Zeno's Paradox and Limits *(5 minutes)*
Understanding Limits: Notation and Graphs	Introduction to and Notation of Limits (8 minutes) Finding Limits on Graph (7 minutes)

Directions: See the HYPERDOC on page 2 for resources on MASTERY & REVIEW of these topics, definitions, and notation. Complete solutions to problems 1 – 3, and then review material in problem 4. Answers - not solutions - are provided at the end, so you may check your work. Make sure you understand the solution behind each answer. Assume that the problems do NOT require a graphing calculator, unless otherwise stated.

1. Let's open with a quick Algebra "correct the mistake" activity.

True or false. If false, change what is underlined to make the statement true.

a. $(x^3)^4 = x^{\underline{12}}$ T F

b. $x^{\frac{1}{2}}x^3 = x^{\underline{\frac{3}{2}}}$ T F

c. $(x+3)^2 = \underline{x^2+9}$ T F

d. $\dfrac{x^2-1}{x-1} = \underline{x}$ T F

e. $(4x+12)^2 = \underline{16}(x+3)^2$ T F

f. $\underline{3}+2\sqrt{x-3} = 5\sqrt{x-3}$ T F

2. Write the equation of a line using the given information. You may use either slope intercept form, $y = mx + b$, or point-slope form $y = m(x - x_1) + y_1$, of the equation of the line.

   a) through the point $(1, 3)$ with slope of -4.

   b) through the points $(1, 3)$ and $(-4, 2)$.

   c) through the point $(1, 3)$ parallel to the line $y = 6(x - 2) + 5$

   d) through the point $(1, 3)$ perpendicular to the line $y = 6(x - 2) + 5$

3. Given the **functions** $f(x) = x^2 - 4x \ \ and \ \ g(x) = 5 - x$, find each of the following:

a) $f(g(-1))$

b) $g(f(-1))$

c) $\frac{f(x)-f(3)}{x-3}$

d) $g^{-1}(-1)$

4. **Factor** the following algebraic expressions completely.

a) $x^2 + 3x + 2$

b) $x^2 + 3x - 4$

c) $x^3 - 3x^2$

d) $x^2 - 9y^2$

e) $sin^2 x - cos^2 x$

f) $2x^2 + 7x + 3$

5. State the **domain** of the following functions.

a) $f(x) = \sqrt{(x+2)}$

b) $g(x) = \frac{x+2}{x^2-4}$

6. Find all solutions to the following equations without a calculator. (Only one-third of the AP exam allows you to use a calculator. These are typical equations you will be solving).

a) $2x - 3 = 5$

b) $\frac{1}{3}x = \frac{5}{12}$

c) $(x + 7)(x - 1) = 0$

d) $5 - (2x - 1) = 10$

e) $x^3 = x^2$

f) $sin(3x) = \frac{1}{2}$, on the interval $[0, 2\pi]$

g) $x^2 + 4x - 21 = 0$

h) $(2x + 3)^2 = 9$

i) $log_3 81 = x$

j) $log_4 x = -3$

7. ***Precalculus Vocabulary you need to know!*** Classifying mathematical objects and assigning them attributes using math vocabulary is an important skill in Calculus. Read each question carefully. Each question is "multiple choice" but you need to know a lot of "math vocabulary words" to answer. If you do not know the meaning of one of the terms please see the resource hyperdoc for a link to "Math is Fun" which has a terrific math dictionary that focuses on visual definitions.

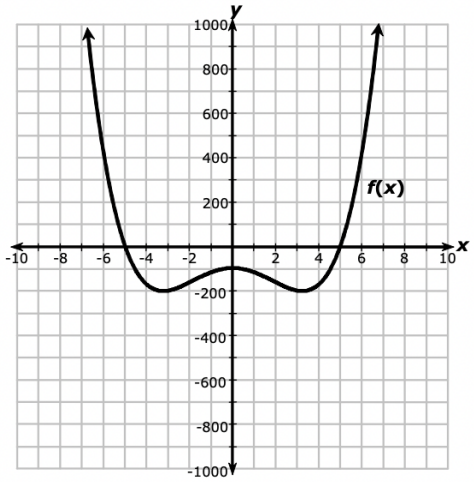
   a. Understanding properties of **Quadratic Functions** is important in Calculus, as the quadratic function models velocity of a falling object.

   Which quadratic function opens upwards and has a vertex at (0 , 3)?

   (A) $y = -(x-3)^2$ (B) $y = (x-3)^2$ (C) $y = -x^2 + 3$ (D) $y = x^2 + 3$

   b. **Polynomial functions** are seen often in Calculus. The statements below contain the term "extrema". Extrema is another way to say "turning points of a graph."

   A polynomial function of degree four is graphed as shown.



   Based on this graph, which statement is true?

   A $f(x)$ has a total of four roots and three local extrema.

   B $f(x)$ has a total of two roots and three local extrema.

   C $f(x)$ has a total of two roots and five extrema.

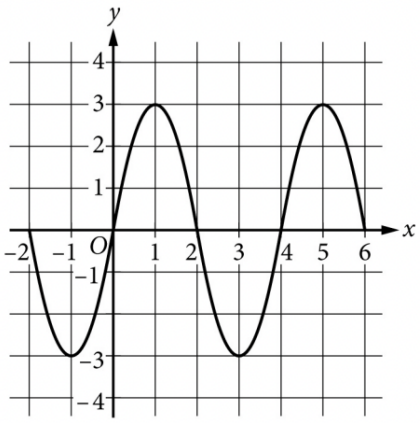
   D $f(x)$ has a total of four roots and five extrema.

c. This question uses “interval notation” in the options. See resource in HYPERDOC to learn about “interval notation”.

What set of intervals describe the domain of the function $f(x) = \frac{x}{x^2+3x}$?

A $(-\infty, -3)\ and\ (-3, \infty)$
B $(-\infty, -3]\ and\ [-3, \infty)$
C $(-\infty, -3],\ [-3, 0)\ and\ (0, \infty)$
D $(-\infty, -3),\ (-3, 0)\ and\ (0, \infty)$

d. Interpreting graphs of functions is an important skill in Calculus. Choose the best response.



Graph of f

Let f be a sinusoidal function. The graph of $y = f(x)$ is given in the xy-plane. What is the period of f ?

(A) 2

(B) 3

(C) 4

(D) 6

e. Relating tables and math models is an important skill in Calculus. This question asks you to decide if a data set is modeled best as a linear or exponential function. Choose the best response.

t (months)	0	1	2	3	4
$P(t)$ (thousands)	20	30	45	67.5	101.25

. The increasing function P gives the number of followers, in thousands, for a new musical group on a social media site. The table gives values of $P(t)$ for selected values of t, in months, since the musical group created their account on this social media site. If a model is constructed to represent these data, which of the following best applies to this situation?

(A) $y = 10t + 20$

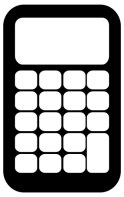
(B) $y = \frac{325}{16}t + 20$

(C) $y = 20\left(\frac{2}{3}\right)^t$

(D) $y = 20\left(\frac{3}{2}\right)^t$

.

8. ***Word Problems.*** Reading carefully and understanding which problem-solving strategy to use is an important skill in Calculus. CALCULATOR ACTIVE!!



a. Eli drops a ball from a height of 6 feet, and the return bounce is 82% of the previous height. How far has the ball traveled up and down when it hits the ground for the sixth time? Round the answer to the nearest tenth of a foot. (hint: A table could help you out)
BONUS: can you write an equation that will predict how far the ball has traveled after x bounces?

b. For an art project, Alex takes a 22 inch by 28 inch poster board and cuts congruent squares out of the corners. She folds the poster board along the cuts to create a container with no top. All responses should include units.
   i. If the size of the side length of the square cut is exactly 3 inches, what is the resulting height of the container?
   ii. What is the resulting volume of the container?
   iii. Write a function, V(x), which returns the volume of the container for a square cut out with side length, x. (hint: draw a picture)

9. ***Difference Quotients.*** Using function notation to write difference quotients is an important skill.

If $f(x) = 3x^2 + 4x$, what is …?

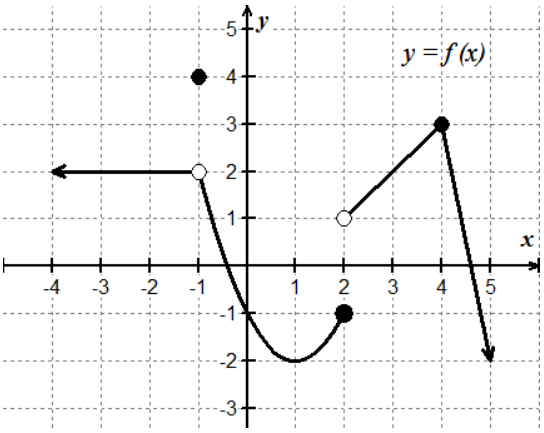
a. $\frac{f(4)-f(1)}{4-1}$

b. $\frac{f(x)-f(1)}{x-1}$

c. $f(x + h)$

d. $\frac{f(x+h)-f(x)}{h}$

10.



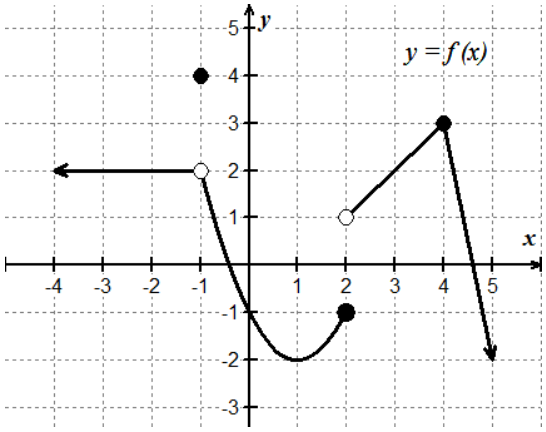
Use the graph above of $y = f(x)$ above to determine the following values.

A. $f(4)$

B. $f(-1)$

C. $f(2)$

11.



After you have watched the videos on graphical limits included in the hyperdoc, answer the following. Use the graph of $y = f(x)$ above to determine the following values [this is the same graph on previous page]. If the limit does not exist, you may write DNE.

a. $\lim_{x \to 4} f(x)$

b. $\lim_{x \to -1} f(x)$

c. $\lim_{x \to 2} f(x)$

d. $\lim_{x \to 2^-} f(x)$

e. $\lim_{x \to 2^+} f(x)$

12. What is the difference between the statements: $\lim_{x \to 4} g(x) = 7$ and $g(4) = 7$?

and finally….

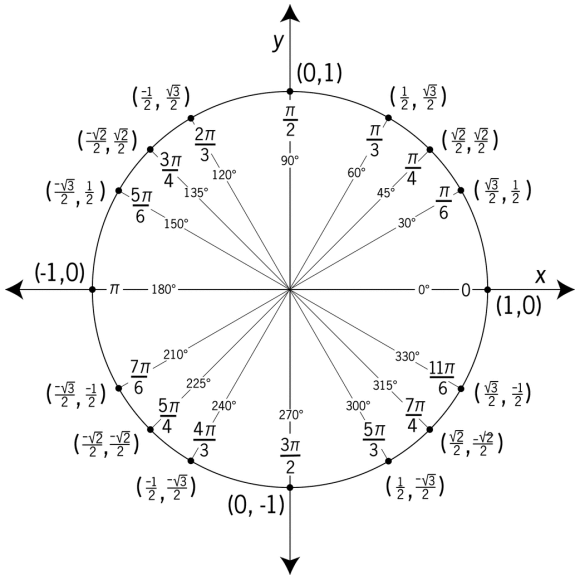
13. Trigonometry Facts Review

Take time this summer to review the values of sine, cosine, and tangent in the unit circle of the "cardinal angles" in radians. You may use whatever method helps you. All the information you are expected to know is in this unit circle.

<u>Having these committed to memory will help calculus run much smoother for you.</u>

Recall that each point on the unit circle $(x, y) = (cos\ \theta,\ sin\ \theta)$ and $x^2 + y^2 = 1$.

θ	0° (or) 0	30° (or) $\frac{\pi}{6}$	45° (or) $\frac{\pi}{4}$	60° (or) $\frac{\pi}{3}$	90° (or) $\frac{\pi}{2}$
sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan θ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Not Defined



WHEW! This is the last of the review problems!

See the next page for the answers - how did you do?

What additional review work should you do?

I am happy to meet with students during pre-season week
or at Camping Trip to go over material! Just reach out!

ANSWER KEY

1. a. True b. False c. False d. False e. True f. False

2. a. $y = -4(x - 1) + 3$ (Point-Slope Form of Linear Equation)
   b. $y = 5(x - 1) + 3$
   c. $y = 6(x - 1) + 3$
   d. $y = -\frac{1}{6}(x - 1) + 3$

3. a. 12
   b. 0
   c. $x - 1$
   d. 6

4. a. $(x + 2)(x - 1)$
   b. $(x + 4)(x - 1)$
   c. $x(x^2 - 3)$
   d. $(x - 3y)(+ 3y)$
   e. $(\sin x - \cos x)(\sin x + \cos x)$
   f. $(x + 3)(2x + 1)$

5. a. $\{x \mid x \geq 2\}$ Roster notation OR $(2, \infty)$ Interval notation
   b. $\{x \mid x \neq -2, 2\}$ Roster notation
   OR $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$ Interval notation

6. a. $x = 4$
   b. $x = \frac{5}{4}$
   c. $x = -7 \text{ or } x = 1$
   d. $x = -2$
   e. $x = 0 \text{ or } x = 1$
   f. $x = \frac{\pi}{18} \text{ or } x = \frac{5\pi}{18}$
   g. $x = -7 \text{ or } x = 3$
   h. $x = -3 \text{ or } x = 0$
   i. $x = 4$
   j. $x = \frac{1}{64}$

7. a. D
   b. B
   c. D
   d. C
   e. D

8. a. 40.4 feet (HINT: total distance traveled is the sum of down and up bounce distances
   b. (i) 3 inches (ii) $704\ in^3$ (iii) $V(x) = x(28 - 2x)(22 - 2x)$

9. a. 7
   b. $3x + 7$
   c. $3(x + h)^2 + 4(x + h) = 3x^2 + 6xh + 3h^2 + 4x + 4h$
   d. $6x + 3h + 4$

10. a. 3
    b. 4
    c. -1

11. a. 3
    b. 2
    c. DNE
    d. -1
    e.. 1

12. answers will vary.

13. n/a