

Multiplication Fact Strategies

2s	<ul style="list-style-type: none">• Double• Add
3s	<ul style="list-style-type: none">• Double + 1 more
4s	<ul style="list-style-type: none">• Double twice
5s	<ul style="list-style-type: none">• Count by 5s• $\times 10 \div 2$
6s	<ul style="list-style-type: none">• $\times 3$ then double
7s	<ul style="list-style-type: none">• $\times 5 + \times 2$
8s	<ul style="list-style-type: none">• Double 3 times
9s	<ul style="list-style-type: none">• finger trick• $\times 10$, - one group
10s	<ul style="list-style-type: none">• stick a 0 on
11s	<ul style="list-style-type: none">• single digit• $\times 10 + 1$ more group
12s	<ul style="list-style-type: none">• $\times 10 + \times 2$

Multiplication Fact Strategies

2s	Double it	Example: $367 \times 2 =$ $\begin{array}{r} 367 \\ + 367 \\ \hline \end{array}$
3s	Double it, then add another of	$7 \times 3 = 21$ $7 \times 2 = 14$ $14 + 7 = 21$
4s	Double Double again	"
5s	x10 then divide by 2	$372 \times 5 =$ $372 \times 10 = 3720$ $3720 \div 2$
6s	140×6 $140 \times 5 = 700$ $700 + 140 = 840$	x3 then double → Do x5 then add the original #
7s		x5, then add the x2...
8s	Double Double Double	$8 \times 7 = 56$ $7 \times 2 = 14$ $14 \times 2 = 28$ $28 \times 2 = 56$
9s	→ Finger trick → Check that the digits add up to 9	x10, then take away one of the original
10s	"add a zero to the end"	
11s	x10 then add one more group	
12s	x10, then add the double	

Decimals Review

Adding and Subtracting

When adding and subtracting decimals, you must:

1. Arrange the numbers so the decimals points line up directly on top of each other.
2. Add or subtract as usual. Be sure to pull the decimal point *directly* down to the answer.

$$\begin{array}{r} 24.5 + 85.4 = 24.5 \\ + 85.4 \\ \hline 109.9 \end{array} \qquad \begin{array}{r} 4.23 - 2.1 = 4.23 \\ - 2.1 \\ \hline 2.13 \end{array}$$

Why?

Lining up the decimals makes sure you are adding and subtracting the correct place values.

Multiplying

When multiplying decimals, you must:

1. Count the total number of digits to the right of the decimal points in both numbers.
2. Multiply both numbers together.
3. Starting from the right, **count** the same number of total decimal places, and insert the decimal point.

$$\begin{array}{r} 1.5 \times .62 = 1.5 \\ \times .62 \\ \hline .930 \end{array}$$

Dividing

When dividing decimals, you must:

1. First, "*get rid of*" the decimal in the **divisor** (the number you are dividing by - see example at right). To do this, move the decimal as many places to the right as necessary to make it a whole number.
2. Move the decimal the same number of places to the right in the dividend (the number being divided - see example at right.) You may need to add zeroes to this number to accomplish this.
3. Divide normally.

Why?

Moving the decimal the same number of places **on both the divisor and the dividend** is the same as multiplying both by the same number: 10 or 100 or 1000 and so on.

$$10.2 \div .51 = \frac{10.2}{.51} = .51 \overline{)10.2}$$

$$\frac{\text{dividend}}{\text{divisor}} = \text{quotient}$$

$$\begin{array}{r} \text{divisor} \quad \text{quotient} \\ \quad \quad .51 \overline{)10.2} \quad \text{dividend} \end{array}$$

The decimal moves two places to the right.

$$\begin{array}{r} 20. \\ .51 \overline{)10.20} \\ \underline{102} \\ 00 \\ \underline{00} \\ 0 \end{array}$$

move this decimal two places to the right. This requires adding a zero.

Different Division

$$\begin{array}{r} 8 \overline{) 729} \\ \underline{- 720} \\ 9 \\ \underline{- 8} \\ 1 \end{array}$$

First, how many 8 in 72? 9. But you are really doing how many times does 8 go into 720, and that would be 90.

Then, you would bring down the answer of 8×90 , and subtract from the number being divided into, 729.

Then, how many times does 8 go into 9? 1.

That would be $91\frac{1}{8}$, or 91.125, because

$$\begin{array}{r} 125 \\ \times 8 \\ \hline 1000 \\ 1000 \\ 1000 \\ 1000 \\ 1000 \\ \hline 10000 \end{array}$$



Polygons

Many Angles
Poly Gons

- Closed figure
- All have straight sides
- 2D
- The sides don't cross

Parallelograms

- Def. A quadrilateral with exactly 2 pairs of parallel sides.

- Features of parallelograms.

- Opposite sides have the same length.
- Opposite angles have the same measure.

- Def. A Rhombus is a quadrilateral with all sides of ~~equal~~ equal length.

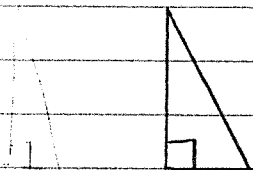
Base — The base of a parallelogram is any side

Height — The height is the measurement at a right angle to the base you chose

Area — The area of a parallelogram can be found by multiplying base times the related height
 $A = b \cdot h$

Triangles

- A triangle is a three-sided polygon
- A polygon is a closed, 2D shape with all straight sides (No curves)
- A right triangle has one right angle



- Features of any triangles:
- Any triangle can be doubled to make a parallelogram
- the Area of a triangle equals half the area of that parallelogram

$$\text{Base} \times \text{height} \div 2 = \text{Area}$$

Triangles

- A closed ^{2D} shape with 3 straight (not curved) sides
- A polygon with 3 sides

Features of a triangle

- No parallel sides
- Any triangle can be doubled to make a parallelogram
- The area of a triangle is half the area of the related parallelogram

Classifying triangles

By sides:

Equilateral: 3 equal sides

Isosceles: 2 equal sides

Scalene: All sides different lengths

By angles:

Right triangle has 1 right angle. (90°)

Obtuse triangle has one angle greater than 90°

Acute triangle: All angles less than 90°

Ratios

The reciprocal of a number is what you multiply it by to get 1.

Dividing by any number is the same as multiplying by its reciprocal.

Multiplying by any non-zero number is the same as dividing by its reciprocal.

Reciprocal of	Is
2	$\frac{1}{2}$
$\frac{3}{5}$	$\frac{5}{3}$

Two ratios are equivalent if you can multiply each of the numbers in the first ratio by the same factor to get the numbers in the second ratio.

Equivalent Ratios

Two ratios are equivalent if you can multiply the numbers in one ratio by the same factor to get the other ratio

Percent

- A percent is a decimal $\times 100$

- Per cent means
per 100 "Out of 100"

Percent strategies

Key: the whole or the "out of" number always pairs with 100%

3 main strategies:

- number lines
- tables
- equations

Number line:

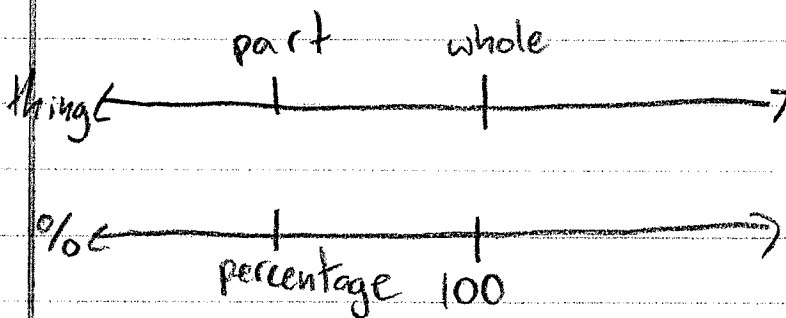


Table:

thing	%
whole	100
part	percentage
	1

Equations:

$$\text{percentage} \times \text{whole} = \text{part}$$

fraction or decimal

$$50\% \times \underline{\quad} = 10$$

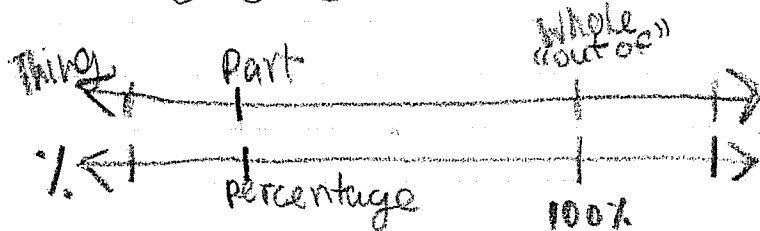
Percent Strategies

Key: The whole or the "out of" number always pairs with 100%.

3 main strategies:

- Number lines
- Tables
- Equations

Number lines



Tables

Thing	Percentage
Whole "out of"	100%
Part	percentage
?	10%
?	1%
?	50%
?	25%
?	20%

Percent Strategies

Equations

$$\text{Percentage} \times \text{Whole} = \text{Part}$$

"out of"

$$75\% \text{ of } 12$$

Fraction of
decimal for
%

$$\left\{ \begin{array}{l} \frac{3}{4} \times 12 \\ .75 \times 12 \end{array} \right.$$

$$\frac{\text{Part}}{\text{Whole}} = \frac{\text{Percentage}}{100}$$

Fractions

- A fraction can be understood as division:
numerator \div denominator = ^{The} Fraction
example = $1 \div 2 = \frac{1}{2}$

- Dividend \div Divisor = Quotient
$$\begin{array}{r} \text{Quotient} \\ \text{Divisor} \overline{) \text{Dividend}} \end{array}$$

Meanings of Divisions

$20 \div 4$ could mean
 \rightarrow 20 split into 4 groups.
 \rightarrow 20 split into groups of 4.

What part of the divisor is the dividend?

or

The dividend is what part of the divisor?

Interpretations of Division

$$\text{Dividend} \div \text{Divisor} = \text{Quotient}$$

~~How~~ How many of these are in that

~~How~~ How many times as large as the ~~divisor~~^{dividend} than the divisor?

☺

Dividing by a fraction:

"Elena's way"

- Multiply the dividend by the denominator of the divisor.
- Divide by the numerator

$$\text{Ex: } 6 \div \frac{3}{4} = 6 \cdot 4 \div 3$$

or

- Multiply the dividend by the reciprocal of the divisor (the 2nd fraction)

Example:

$$6 \div \frac{3}{4} = 6 \cdot \frac{4}{3} = \frac{6}{1} \cdot \frac{4}{3}$$

$$= \frac{6 \cdot 4}{1 \cdot 3} = \frac{24}{3} = 8$$

Fractions in a Nutshell

What fractions are: A fraction is a number that expresses its value as parts of a whole. The denominator (bottom number) tells how many parts would make one whole. The numerator (top number) tells how many parts you actually have.

Example: $\frac{5}{8}$ means 8 parts make up one whole, and you have 5 of those parts. If a pizza is cut into 8 slices and you have 5 of them, you have $\frac{5}{8}$ of one whole pizza.

2. What else fractions are: A fraction is also a division problem. The numerator (top) is divided by the denominator. Use this fact to convert fractions to decimals.

Example: $\frac{5}{8} = 5 \div 8 = .625$

To convert the other way around (make a decimal into a fraction), use the place values of the decimal to write it as a fraction. Then reduce (write the fraction in lowest terms).

Example: .625 is "six hundred twenty-five thousandths."
Therefore $.625 = \frac{625}{1000}$. Now reduce: $\frac{625}{1000} = \frac{125}{200} = \frac{5}{8}$

3. Writing in lowest terms: Divide the numerator (top) and denominator (bottom) by the SAME number. Repeat this until there are no more numbers that go into both the top & bottom evenly.

Ex: $\frac{2}{4} = \frac{1}{2}$

$\frac{12}{6} = \frac{6}{3} = \frac{2}{1} = 2$ etc.
Divide by 2 Divide by 3

4. Improper fractions & mixed numbers: Divide the denominator of an improper fraction into the numerator. Then write the answer, with the remainder written over the same denominator.

$$\text{Ex: } \frac{7}{2} \quad \begin{array}{r} 3R1 \\ 2 \overline{)7} \end{array} \quad \text{so } \frac{7}{2} = 3\frac{1}{2}$$

For a mixed number, multiply the denominator by the whole number, then add the numerator. Write this number over the denominator.

$$\text{Ex: } 3\frac{1}{2} \quad (2 \times 3) + 1 = 7 \quad \text{so } 3\frac{1}{2} = \frac{7}{2}$$

5. Adding: Rewrite the fractions so that they have the same denominator.

The new fractions must still equal the original fractions.

To obtain new fractions that are equal to the original ones, you must multiply the numerator and denominator by the same number. Then add up the numerators of the new fractions, and leave the denominator unchanged. Reduce your answer if necessary.

$$\text{Ex: } \begin{array}{r} \frac{5}{6} = \frac{5}{6} \\ + \frac{2}{3} = \frac{4}{6} \quad (\text{multiply top +} \\ \quad \quad \quad \text{bottom by 2}) \\ \hline \boxed{\frac{9}{6}} \end{array}$$

$$\text{Ex. } \begin{array}{r} 2\frac{1}{4} = 2\frac{3}{12} \quad (\text{multiply top +} \\ \quad \quad \quad \text{bottom by 3}) \\ + 3\frac{1}{12} = 3\frac{1}{12} \\ \hline 5\frac{4}{12} = \boxed{5\frac{1}{3}} \end{array}$$

6. Subtracting - Follow all the same steps as for adding, but subtract one numerator from the other.

$$\text{Ex: } \begin{array}{r} \frac{5}{6} = \frac{5}{6} \\ - \frac{2}{3} = \frac{4}{6} \\ \hline \boxed{\frac{1}{6}} \end{array}$$

$$\text{Ex. } \begin{array}{r} 8\frac{1}{2} = 8\frac{7}{14} \\ - 3\frac{1}{7} = 3\frac{2}{14} \\ \hline \boxed{5\frac{5}{14}} \end{array}$$

7. Multiplying: To multiply fractions, just multiply "straight across." That is, multiply numerator times numerator, and denominator times denominator. (If you have a mixed number, you must change it to an improper fraction before you can multiply. See comment #4.)

$$\text{Ex. } \frac{8}{15} \times \frac{3}{16} = \frac{8 \times 3}{15 \times 16} = \frac{24}{240} = \frac{1}{10}$$

Note that you can make problems MUCH simpler by cancelling as much as possible before you multiply. Do this by dividing a number on the top and a number on the bottom by the same number. We can use this on the problem in the above example:

$$\frac{\overset{1}{\cancel{8}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{8}{\cancel{16}}} = \frac{1}{5} \times \frac{1}{8} = \frac{1}{40}$$

Example with mixed numbers:

$$4\frac{1}{2} \times 5\frac{1}{3} = \frac{9}{2} \times \frac{16}{3} = \frac{24}{1} = 24$$

8. Dividing: To do a fraction division problem, change the problem to multiplication, and change the second fraction into its reciprocal. More simply, FLIP THE SECOND ONE and multiply.

Examples:

$$\frac{3}{4} \div \frac{1}{6} = \frac{3}{4} \times \frac{6}{1} = \frac{9}{2}$$

$$1\frac{1}{8} \div \frac{3}{7} = \frac{9}{8} \div \frac{3}{7} = \frac{9}{8} \times \frac{7}{3} = \frac{21}{8}$$

$$4\frac{5}{6} \div 3\frac{2}{9} = \frac{29}{6} \div \frac{29}{9} = \frac{29}{6} \times \frac{9}{29} = \frac{3}{2}$$

Order Of Operations

G^oP ① Do any work with grouping symbols

E ② Do any Exponents

\overrightarrow{MD} ③ Do all \times and \div in order from left to right

\overrightarrow{AS} ④ Do all $+$ and $-$ in order from left to right

example

$$60 \div 3 \cdot 2$$

$$20 \cdot 2$$

$$\boxed{40}$$

Expressions & Equations

An equation is a math sentence with an ~~math~~ Equals sign. $=$

The equals sign means that everything on one side has the exact same value as everything on the other.

$$5 + 1 = 2 + 3$$

A variable is a letter that stands for an "unknown" number.

Inverse operations

"undo" each other.

$+$ is the inverse of $-$

\times is the inverse of \div

Coefficient: A number written directly in front of a variable. The variable is multiplied by that number.

Solution - A number that makes an equation true when you substitute it in for the variable

Solving Equations

you can solve

$$\cancel{ax} a \cdot x = b \text{ for } x$$

by dividing both sides by a .

$$\frac{1}{2} \cdot x = \frac{3}{8}$$

$$\div \frac{1}{2} \quad \div \frac{1}{2}$$

$$x = \frac{3}{8} \div \frac{1}{2} = \frac{3}{4}$$

you can solve

$$c + y = d \text{ for } y$$

by subtracting c from both sides.

$$.5 + y = .7$$

$$\underline{-.5} \quad \underline{-.5}$$

$$y = .2$$

SOLVING EQUATIONS

You can solve:

$A \cdot X = B$ for x : by dividing both sides by a

Solve: find the solution

Solution: a number that ~~makes~~ makes the Equation true

You can solve:

$C + y = d$ for y : by subtracting c from both sides

Rational Numbers

A rational number is any number that can be written as a fraction (or ratio) = /

Ex. 1, .5, $\frac{1}{2}$, .75, 72, $\bar{3}$, $-\frac{2}{3}$, .123784, 0

They are all whole numbers, terminating decimals, and all repeating decimals, whether positive or negative.

Irrational Numbers

A non-terminating, non-repeating decimals

Ex. π $\sqrt{3}$

Absolute Value

of any number is its distance from 0 on a number line.

Ex: $|5| = 5$

$|-12| = 12$

$|0| = 0$

Opposites

in math are two numbers that are the same distance from 0.

Ex: 12 and -12

-2.1 and 2.1

The symbol for "the opposite of" is "-"

"-a" is NOT "negative a"

"-a" is "the OPPOSITE of a"

Factors

are whole numbers that can be multiplied to result in a given product.

Ex: Factors of 8 are

1, 2, 4, 8

The Greatest Common Factor of 2 (or more) numbers is the biggest number that is a factor of both (or all) the numbers.

A multiple of a number is a number you can find by skip-counting or multiplying by a whole number.

Ex: Multiples of 3 include

3, 6, 9, 12, 15 ----- $\rightarrow \infty$

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